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DESTRUCTION OF THE STRETCHED ROD WITH NON-UNIFORM TEMPERATURE FIELD FROM THE EFFECT OF AGGRESSIVE MEDIUM

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A long-term strength of the rod made of the damaging material with non-uniform temperature field at pulling in the aggressive medium has been investigated. A combined effect of the aggressive contact medium and non-uniform temperature distribution on dispersing destruction have been studied. It is shown that such an effect reduces the destruction time.

Key words: rod, damaging, dispersing destruction, aggressive medium, temperature.

Introduction. Experience shows that the medium of aggressive nature exert debilitative effect on the constructive materials. This effect basically causes a reduction in the strength limit. Thus, strength limit σ_{Π} depends on the concentration c of the particles of aggressive medium diffused into structural material. Such dependence has been accepted elsewhere [1] in the form of the linear approximation dependence:

$$\sigma_{\Pi} = \sigma_0 (1 - \gamma c) \tag{1}$$

where γ is the proportionality coefficient determined from experience.

As problem is solved in uniaxial case, the wall can be considered as a rod. If we take the rod thickness equal *2h*, and direct *x*-axis perpendicular, *y*-axis along the rod axis, then aggressive medium concentration satisfying diffusion equation in stationary case can be accepted in following simple form:

$$c(x) = \frac{h - x}{2h} \tag{2}$$

It is assumed that aggressive medium is located in $x = -h$ side. According to (2) boundary conditions $c(-h) = 1; \quad c(h) = 0$ are satisfied.

The effect of aggressive medium on construction material is assumed to be small. As material is undergone to destruction and deformation, its strength and deformation is determined by genetic-type destruction theory [3].

In the same case side faces of the rod have constant temperature and as a result temperature in rod is distributed heterogeneous throughout the thickness of the rod.

Problem statement: Suppose that the side surface of the rod of *2h* width whish is under the effect of constant traction force *P* is under effect of constant, different temperatures T_1 and $T_2(T_2 > T_1)$ (Figure 1). Let's investigate the dispersed destruction process for this rod. Let's accept that the temperature field across the rod varies in linear fashion:

$$\begin{cases} T(x) = a + bx \\ a = \dfrac{T_2 + T_1}{2}; \quad b = \dfrac{T_2 - T_1}{2h} \end{cases} \tag{3}$$

The issue is solved for the case of plane deformation. It is assumed that all the parameters involved in the solution of the problem depend only on *x*-coordinate.

Balance equation is written:

$$\begin{cases} \sigma_{xx,y} + \sigma_{xy,y} = 0 \\ \sigma_{xy,x} + \sigma_{yy,y} = 0 \end{cases} \tag{4}$$

From this system easily $\sigma_{xx} = c_1;\ \sigma_{xy} = c_2$ is obtained, where c_1 and c_2 are arbitrary constants. Since on the rod surface $\sigma_{xx}\big|_{x=\pm h} = \sigma_{xy}\big|_{x=\pm h} = 0$, we obtain $c_1 = c_2 = 0$. As a result, despite the tension $\sigma_{xx} = 0$ and $\sigma_{xy} = 0$ do dot depend on transverse coordinates the rod bending does not considered. It is believed that high tension stress does not allow rods bending.

Thus among the tension components only σ_{yy} is non-zero, i.e. tension state is uniaxial. Then

$$\sigma_{yy} = \sigma - \chi T \tag{5}$$

where σ is uniaxial tensile tension and $\sigma = const; \chi T$ - thermal stress; $\chi = \alpha E$ - constant, a - coefficient of linear expansion; E - coefficient of elasticity.

According to equilibrium equation we obtain:

$$\int_{-h}^{h} \sigma_{YY} dx = P \tag{6}$$

given here (5), we obtain:

$$\int_{-h}^{h} (\sigma - \chi T) dx = P \tag{7}$$

Given that $\sigma = const$, from (7) is easily obtained:

$$\sigma = \frac{p}{2h} + \frac{\chi}{2h}\int_{-h}^{h} T dx \tag{8}$$

Inserting (8) into (5), we obtain the following expression for σ_{yy}:

$$\sigma_{yy} = \frac{p}{2h} + \frac{\chi}{2h}\int_{-h}^{h} T dx - \chi T \tag{9}$$

To investigate the first incubation step of the destruction the following destruction criterion is used:

$$\sigma_{\max} + M^{\bullet} \cdot \sigma_{\max} = \sigma_0 (1 - \gamma c) \tag{10}$$

where γ is proportionality coefficient of the aggressive medium.

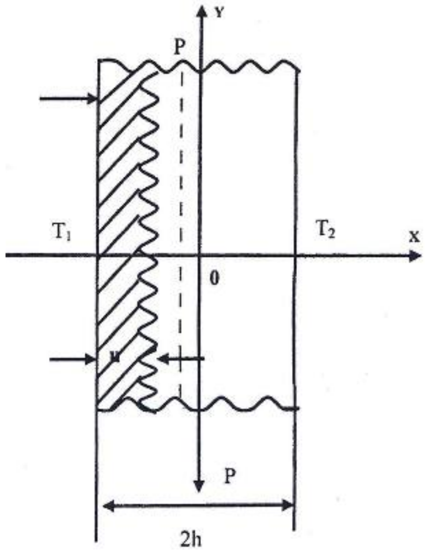

Fig. 1. Longitudinal cross-section of the rod elongated in a non-uniform temperature field.

Here - $M^{\bullet}$ is integral damage operator of genetic type that behaves in the state of monotonically changing stress as a normal elastic operator; σ_0 - a strength limit of the material without damages.

Solution of the problem. Taking into account the temperature distribution (3) in (9), we can write for σ_{yy}:

$$\sigma_{yy} = \frac{p}{2h} - \chi b x; \qquad b > 0 \tag{11}$$

It is seen from this expression, that maximum value of the tension σ_{yy} is reached in the border $x = -h$ and initial destruction occurs in this section:

$$\sigma_{\max} = \sigma_{yy}\Big|_{x=-h} = \frac{p}{2h} + \chi b h \tag{12}$$

Inserting (12) into destruction criterion (10), we obtain for the beginning moment of the destruction or incubation period:

$$\int_0^{t_0} M(\tau) = \frac{\sigma_0(1-\gamma)}{\sigma_{\max}} - 1 \qquad (13)$$

If kernel $M(t)$ is $M(t) = m = const$, from (13) we obtain:

$$mt_0 = \frac{2h\sigma_0(1-\gamma)}{P + \chi h\Delta T} - 1; \qquad \Delta T = T_2 - T_1 \qquad (14)$$

From (14) for latent period of the destruction it is found:

$$P < P_0 = h(2\sigma_0(1-\gamma) - \chi\Delta T) \qquad (15)$$

When the tractive force is $P > P_0$, destruction occurs momentarily. It is seen from the expression (15) that under the effect of the temperature field the boundary value for the force P_0 decreases. And from (14) reduction of the incubation period is obtained. The presence of the aggressive medium reduces the value of the limiting force P_0.

After the destruction of $x = -h$ surface the destruction surface moves to the next surface, and after some time τ destructed layer of width u is created (Figure 1). The width of non-destroyed layer of the rod equals h-u. In this case the tension at the point $x = -(h-\upsilon)$, $(\upsilon > u)$ is determined from (5) by substituting the lower limit of integration in the expression (7) by $-h$-1– $(h-u)$ and the result is:

$$\sigma_{yy}\Big|_{x=-(h-u)} = \frac{1}{2h-u}\left(P + \chi \int_{-(h-u)}^{h} T dx\right) - \chi T \qquad (16)$$

Let's assume that at the moment $t > \tau$ the width υ of the destroyed part if a function of time, i.e. $\upsilon = u(t)$. Then expression (16) can be written as follows:

$$\sigma_{yy}(t,\tau) = \frac{1}{2h - u(\tau)}\left(P + \chi \int_{-h+u(\tau)}^{h} T dx\right) - \chi T\Big|_{x=-h+u(t)} \qquad (17)$$

Inserting into (17) the expression (3) for the temperature we obtain:

$$\sigma_{yy}(t,\tau) = \frac{P}{2h - u(\tau)} + \chi b\left(h + \frac{1}{2}u(\tau) - u(t)\right) \qquad (18)$$

If this expression to take into account in the destruction criterion written for the destruction front, we obtain the following expression: $(x = -h + u(t))$

$$\sigma_{yy}(t,t) + \int_0^t M(t-\tau)\sigma_{yy}(t,\tau)d\tau = \sigma_0(1 - \gamma c(t)) \qquad (19)$$

If to insert (18) and (2) into (19), then for the function $u = u(t)$ characterizing the width of the destructed part, we obtain the equation:

$$\frac{P}{2h-u(t)}-\frac{\chi b}{2}\left[2h-u(t)\right]+\chi b\left[h-u(t)\right]\cdot\int_0^t M(\tau)d\tau+$$

$$+\int_0^t M(t-\tau)\left\{\frac{P}{2h-u(t)}-\frac{\chi bu(t)}{2}\right\}d\tau=\sigma_0\left(1-\gamma\frac{2h-u(t)}{2h}\right) \qquad (20)$$

Equation (20) is non-linear integral equation and it is solved numerically. In the particular case when kernel of destruction operator is constant, i.e. $M(t)=m=const$, let's simplify the expression (20):

$$\frac{P}{2h-u(t)}-\frac{\chi b}{2}\left[2h-u(t)\right]+m\int_0^t\left(\frac{P}{2h-u(\tau)}-\frac{\chi b}{2}u(\tau)\right)d\tau+$$
$$+\chi bm(h-u(t))t=\sigma_0\left(1-\gamma\frac{2h-u(t)}{2h}\right) \qquad (21)$$

Differentiating the expression (21) relative to the time we obtain the following expression with respect to derivative of the desired function *u(t)*:

$$u^{\bullet}=m\frac{P(2h-u)^{-1}+\chi b(h-1{,}5u)}{P(2h-u)^{-2}+\chi b(0{,}5+mt)+\frac{\sigma_0\gamma}{2h}} \qquad (22)$$

Initial condition for the resulting differential equation will be as following:

$$u\big|_{t=t_0}=0 \qquad (23)$$

Note that the expression (22) is the expression for the velocity of propagation of the destruction front. The function $\dot{u}$ at any time, including the initial moment, is positive:

$$\dot{u}\big|_{t=t_0}=2mh\frac{P+2h^2\chi b}{P+2h^2\chi b(1+2mt_0)+2h\sigma_0\gamma} \qquad (24)$$

A denominator on the right-hand side of equation (22) never vanishes. To solve this equation numerically it is reasonable to rewrite it in dimensionless quantities. We introduce the following dimensionless quantities:

$$\begin{cases} f=\frac{u}{h}; \quad \theta=mt; \quad \beta=\frac{h^2\chi b}{P} \\ f^{/}=\frac{df}{d\theta}; \ \omega=\frac{2h\sigma_0}{P} \end{cases} \qquad (25)$$

With these variables, equation (22) takes the form:

$$f' = \frac{(2-f)^{-1} + \beta(1-1{,}5f)}{(2-f)^{-2} + \beta(0{,}5+\theta) + 2{,}25\omega^{-1}\gamma} \quad (26)$$

The initial condition (23) will be as follows:

$$f\Big|_{\theta=\theta_0} = 0 \quad (27)$$

In accordance with (14) the time T_0 will be as follows:

$$\theta_0 = \frac{\omega(1-\gamma)}{1+2\beta} - 1 \quad (28)$$

The expression for the velocity of the destruction front in dimensionless variables has the following form:

$$f'\Big|_{\theta=\theta_0} = 2\frac{1+2\beta}{1+2\beta(1+2\theta_0)+\omega^{-1}\gamma} \quad (29)$$

Numerical calculations. In Figure 2 initial destruction time plot against proportionality coefficient of the aggressive medium according to (28) is presented.

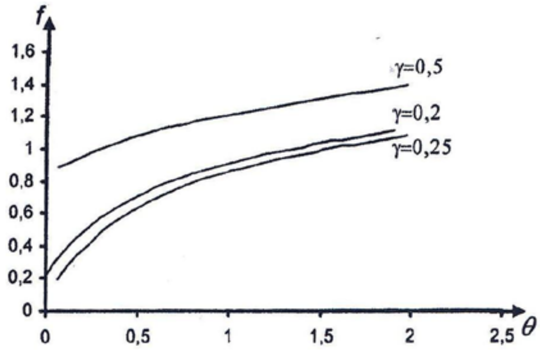

Fig. 2. Dependence of the incubation period on the coefficient of proportionality of the aggressive medium.

Figure 3 shows the graphics of the destruction front propagation corresponding to different values of the γ coefficient characterizing aggressive medium $(\gamma = 0;\ 0.025;\ 0.05....0.2)$.

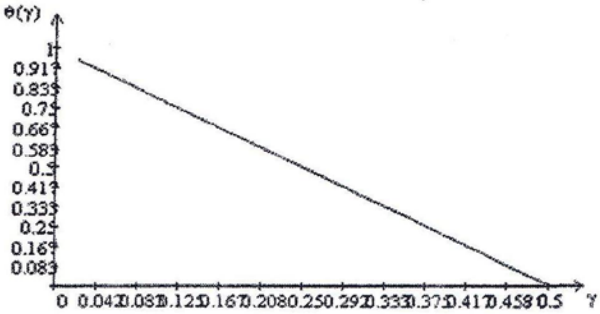

Fig. 3. Curves of the destruction front propagation.

Conclusion. A combined effect of the aggressive contact medium and non-uniform temperature distribution on dispersing destruction has been studied. It is shown that such an effect reduces the destruction time.

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QEYRİ-BİRCİNS TEMPERATUR SAHƏLİ DARTILAN ÇUBUĞUN AQRESSİV MÜHİT TƏSİRİ ALTINDA DAĞILMASI

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XÜLASƏ

Zədələnən materialdan hazırlanmış qeyri-bircins temperatur sahəsinə malik çubuğun aqressiv mühitdə dartılmasında uzunmüddətli möhkəmliyi tədqiq edilmişdir. Aqressiv mühitin səpələnmiş dağılmaya təsiri temperaturun qeyri-bircins paylanması ilə birlikdə öyrənilmişdir. Bu təsirin dağılma zamanının azalmasına gətirməsi göstərilmişdir.

Açar sözlər: çubuq, zədələnmə, səpələnmiş dağılma, aqressiv mühit, temperatur.

ПОВРЕЖДАЕМОСТЬ РАСТЯЖЕННОГО СТЕРЖНЯ С НЕОДНОРОДНЫМ ПОЛЕМ ТЕМПЕРАТУРЫ ПОД ВЛИЯНИЕМ АГРЕССИВНОЙ СРЕДЫ

Э.М.МУСТАФАЕВА

РЕЗЮМЕ

Исследована длительная прочность стержня в агрессивной среде, изготовленного из повреждающегося материала при расстяжение в неоднородной температуре. Действие агрессивной среды на рассеянное разрушение изучено совместно с действием неоднородного распростронения температуры. Показан привод уменьшения времени разрушения этого действия.

Ключевые слова: стержень, повреждаемость, рассеянное разрушение, агрессивная среда, температура.

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